

QUAD TREE SEGMENTATION OF SPECULAR IMAGERY  
VIA BESOV SPACE MERGE CRITERION

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ABSTRACT

Certain specular types of sensor images (e.g., laser) contain vital information which is difficult to glean from non-specular sources. The present and increasing deluge of these types of images has created a critical need for image processing algorithms which reduce the workload for the image analyst by performing some of his/her functions automatically. Many of these algorithms are based on *image segmentation*—a procedure having 1.) high separability between target object and background and 2.) low computational-intensity implementation as two key goals. A large number of algorithms for automatic segmentation of images have been tendered, most occurring within the *Mumford-Shah* paradigm which uses approximation error, boundary length, and variance as weighted terms in an energy functional. The new idea of the present paper is to generalize the Mumford-Shah variance energy so that it directly measures the relative smoothness memberships of target and object background. This is especially important in the application to segmentation of specular types of images which tend to require the separation of subtle grades of smoothness and the unraveling of delicate smoothness space interpolations. The fast wavelet transform answers the purposes of efficient determination of smoothness membership at global as well as local levels and works well in a quad tree architecture.

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## 1.0 FAST, AUTOMATIC IMAGE SEGMENTATION

In sensors employing wavelengths comparable to the physical dimensions of the object or background being imaged, bright, crest-on-crest and dark trough-on-trough interference features, called speckle, figure prominently. In most instances, the presence of these happenstance reinforcements and cancellations complicates the extraction of application-important objects present in the image. Still, certain specular types of sensor images contain vital information which is difficult or impossible to glean from the non-specular imaging systems. Further, the greater numbers of sensor systems being deployed, and improvements to the resolution of these systems, have created a deluge of images for image analysts to cope with. Consequently, there is a critical need for image processing algorithms which reduce the workload for the image analyst by performing some of his/her functions automatically.

An example automatic, fast algorithm, based on segmentation and designed to assist the image analyst, is Northrop Grumman Corporation's Early Vision Image Enhancement, depicted in Figure 1. The image is segmented into regions differing in brightness in order to permit the localization of gray-level assignments within these. This provides a better informational channel match at the "gray-scale assignment channel" so that higher rates of semantic information may be conveyed to the viewer<sup>4</sup>.

An enormous number of algorithms for automatic image segmentation have been developed. Most of these may be characterized as various types of Mumford-Shah segmentations<sup>5</sup>. Typically, the split and merge operations driven by the Mumford-Shah energies are performed in a convenient tree algorithm where operations at coarse, global levels greatly reduce the computational intensity for the fine, local levels. The classic *quad tree* segmentation algorithm is an important benchmark coarse-to-fine architecture for performing Mumford-Shah segmentation<sup>6</sup>.

In particular, the Mumford-Shah model seeks, given an image  $g(x)$ , a piecewise smoothed image  $u(x)$  with a set  $K$  of abrupt discontinuities, the edge set of the segmentation. This is sought by minimizing the functional

$$E(u, K) = \int_{\Omega \setminus K} [|\nabla u(x)|^2 + (u-g)^2] dx + \text{length}(K) \quad \text{Eq. 1.}$$

Where  $\Omega \setminus K$  is the image domain sans boundary set<sup>5</sup>.

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4. The problem of conveying semantic information across a communication channel is the "Level B" problem referred to in The Mathematical Theory of Communication, Shannon and Weaver, University of Illinois Press, 1949.

5. Jean-Michel Morel and Sergio Solimini, Variational Methods in Image Segmentation, Birkhauser, 1995. (See also: D. Mumford and J. Shah, "Optimal Approximations by Piecewise Smooth Functions and Associated Variational Problems," Communications on Pure and Applied Mathematics, vol. XLII No. 4, 1989.)

6. Bart M. ter Haar Romeny (Ed.), Geometry-Driven Diffusion in Computer Vision, Kluwer, 1994.

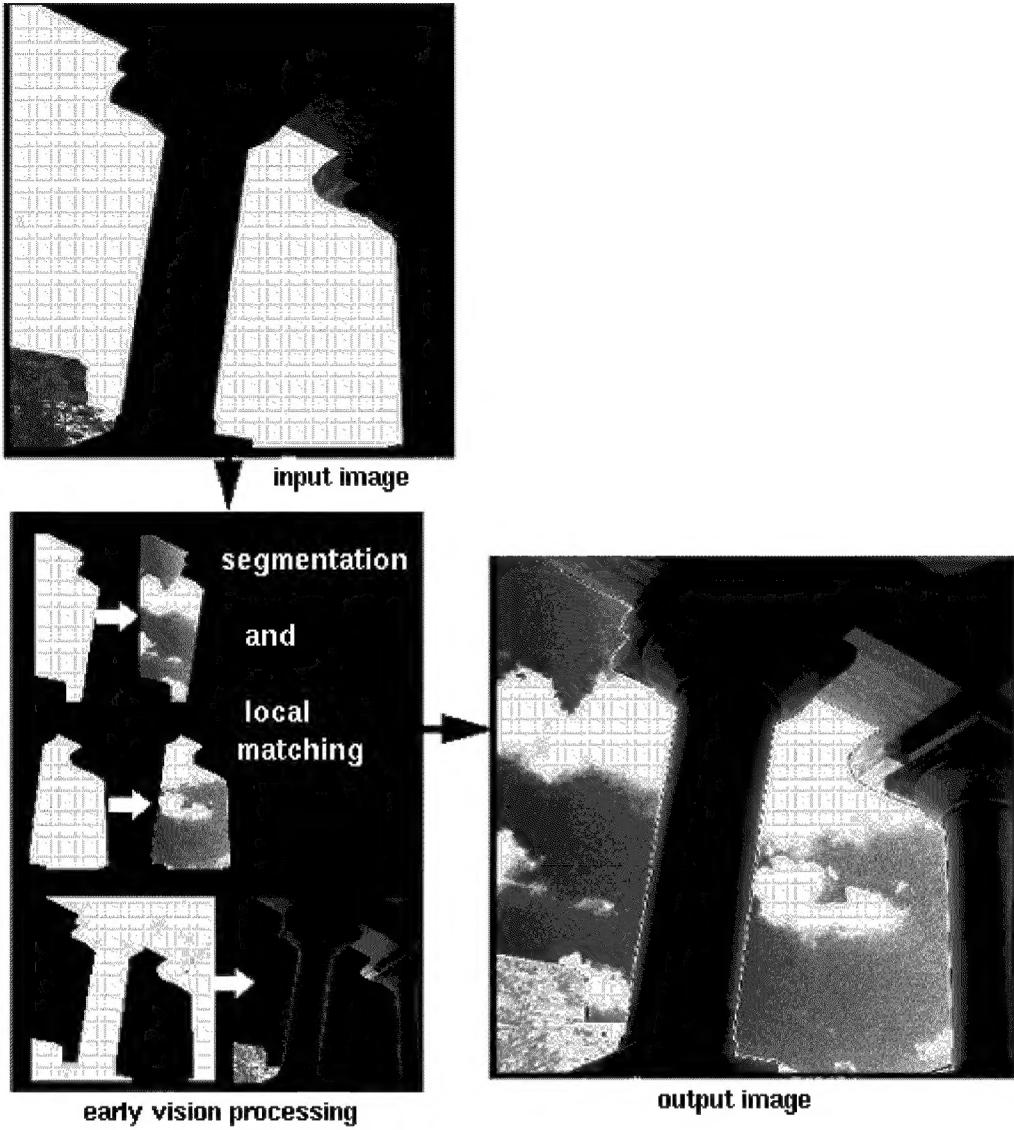


Figure 1. Early Vision Image Enhancement, an example image segmentation application.

An ordinary generalization of Eq. 1 expresses the Mumford-Shah energy as the sum of three terms—the *variance energy*, the *piecewise approximation error energy*, and the *edge length energy*:

$$E_{\text{mumford-shah}} = E_{\text{variance}} + E_{\text{piecewise approximation}} + E_{\text{edge length}} \quad \text{Eq. 2}$$

In a great many applications (perhaps most applications) the second term is zero because raw data compose the segment interiors (apart from contrast and brightness adjustments) making  $\int (u - g)^2 dx = 0$ . The present paper is geared toward segmentations of this type.

## 2.0 ADAPTING MUMFORD-SHAH TO THE SPECULAR PROBLEM

Managing the relative importance of length energy and smoothness energy is a critical issue for specular types of images. This manifests as a tendency for the segmentation to “shatter” as the algorithm proceeds

from large-scale to small-scale split-and-merge operations. An example shattered segmentation is illustrated in Figure 2. The stucco wall in this “pedestrian” image approximates the appearance of speckle. The increasing difficulty in separating object and background as the smaller scale, more-local levels are approached (small-scale shattering) is typical of specular images.

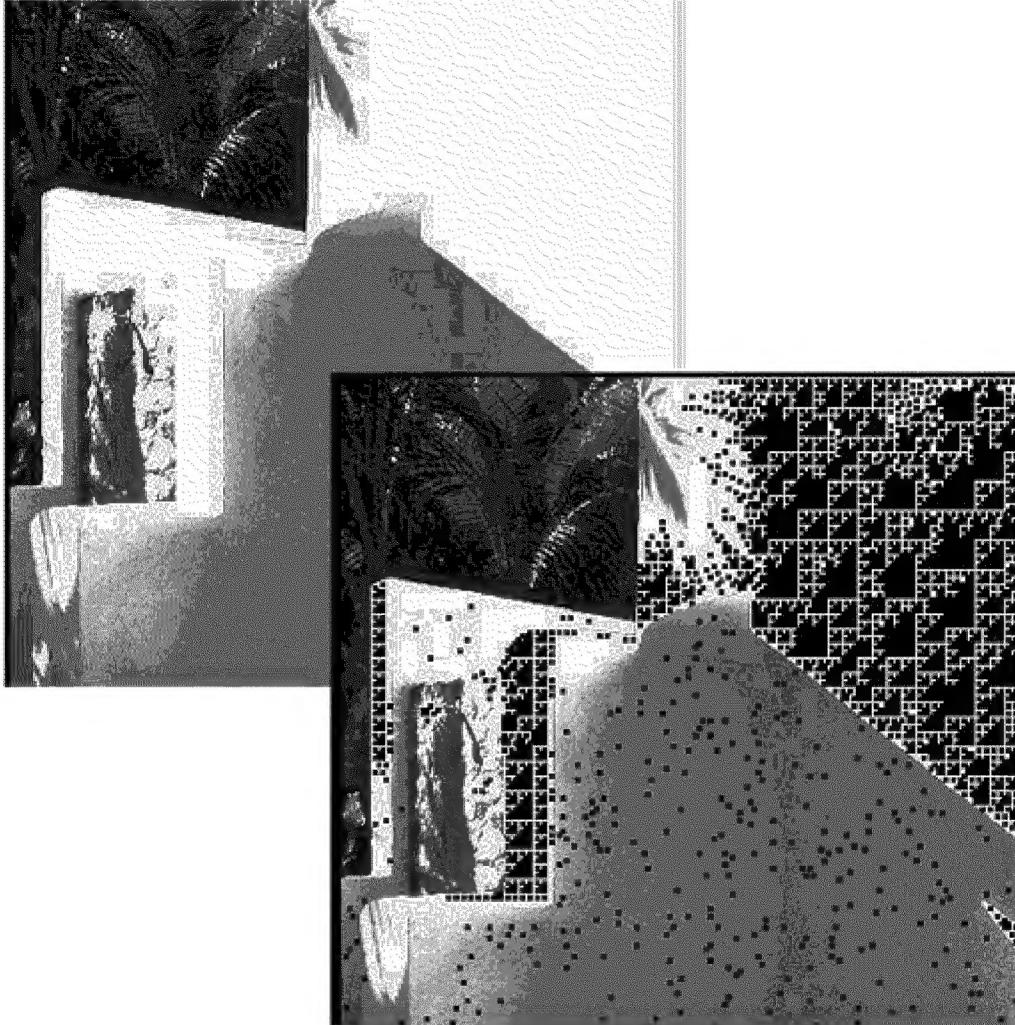


Figure 2. An image segmentation which has “shattered” owing to the presence of stucco texture (which answers the purpose of simulating speckle for this “pedestrian” image).

Toward maintaining greater separability between object and background, so as to provide against or reduce the phenomenon of segmentation shattering, we propose a slight generalization of the Mumford-Shah energy, involving some recent interpolation space ideas. In particular, the purpose of maintaining high separability between object and background is more directly answered by replacing the variance energy  $E_{\text{variance}}$  with a “smoothness separation” energy  $E_{\text{relative separation}}$  which measures the relative smoothness space membership of object and background during segmentation:

$$E_{\text{mumford-shah}} = E_{\text{relative smoothness}} + E_{\text{edge length}} \quad \text{Eq. 3.}$$

In particular, we propose the use of the *interpolation functor* for the interpolation between the Besov space of the object of interest and the Besov space of the image background as the  $E_{\text{relative smoothness}}$  term for the Mumford-Shah segmentation.

Besov spaces are small enough (i.e., special enough) to characterize in great depth much of what is important about an image. For example, in image compression, the approximation error versus the compressed size follows a power law given by the Besov membership of the image<sup>7</sup>. On the other hand, Besov spaces are large enough (i.e., general enough) to hold within their compass the great bulk of image types of interest to applications. Performing segmentation so as to maintain separate smoothness memberships locally (in terms of Besov space membership) during segmentation extends the Mumford-Shah theory in an important way. No longer limited to segmentation into “smooth” versus “coarse” segments, we now possess a function-analytic framework for pursuing segmentations between regions possessing more subtle differences in smoothness. Further, the connections to the interpolation spaces theory<sup>8</sup> championed by Peetre and many others allow precise statements concerning the sense in which a segmentation is optimal.

Owing to the work of DeVore et al<sup>7</sup>, the Besov space membership may be estimated locally or globally in a very small number of operations using orthonormal wavelets. Thus the separability offered by the relative Besov space memberships comes with fast algorithms at all scales during a segmentation. The parthenon image along with its decomposition into orthonormal, Daubechies wavelets (via the Mallat decomposition) is presented in Figure 3. The L1 and L2 energies as a function of scale are also shown.

A theorem of DeVore, Jaweth, and Lucier provides that, given an image  $f \in L_p(\mathbb{R}^2)$ , and given a multiresolution analysis  $V_j$  with wavelet  $\psi$ , in order for  $f$  to belong to Besov space  $B_q^\alpha(L_p)$  it is necessary that:

$$\left\{ \sum_j (2^{j\alpha p} \sum_k 2^{j(p-2)} |c_{k,j,\psi}|^p)^{q/p} \right\}^{1/q} < \infty \quad \text{Eq. 4.}$$

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7.Ronald A. DeVore, Bjorn Jawerth, and Bradley J. Lucier, “Image Compression Through Wavelet Transform Coding,” IEEE Trans. Info. Theo., VOL. 38, NO.2, March 1992.

8.J. Bergh and J. Lofstrom, Interpolation Spaces, Springer-Verlag, 1976.

Where scale is  $2^j$  in the multiresolution  $V_j$ ,  $k$  is the lattice point  $k \equiv (k_1 \bullet 2^j, k_2 \bullet 2^j)$ ,  $k_1, k_2 \in \mathbb{Z}$ ,  $\alpha$  is the Besov “distributional derivative,”  $q$  is the fine smoothness such that  $1/q = \alpha/2 + 1/p$ , and where also  $f = \sum_{k,j} c_{k,j,\psi} \psi_{k,j}$  is the representation in the orthonormal wavelets  $\psi_{k,j}(\varphi) = 2^j \psi(2^j \varphi - j)$  in wavelet coefficients  $c_{k,j,\psi}$ .

Eq. 4 is the interpolation functor between the Besov space to which  $f$  belongs and  $L_p$ . The wavelet transform, offering a dyadic architecture, permits the estimation of the Besov space functor locally, at each stage of a quad tree segmentation. In our approach, the  $E_{\text{relative smoothness}}$  is proportional to the difference between the  $K$ -functors for the object and background at each level of resolution. Candidate regions tend to be broken off into separate segments when their Besov membership differs greatly from that of the object as balanced against the impact of the length energy.

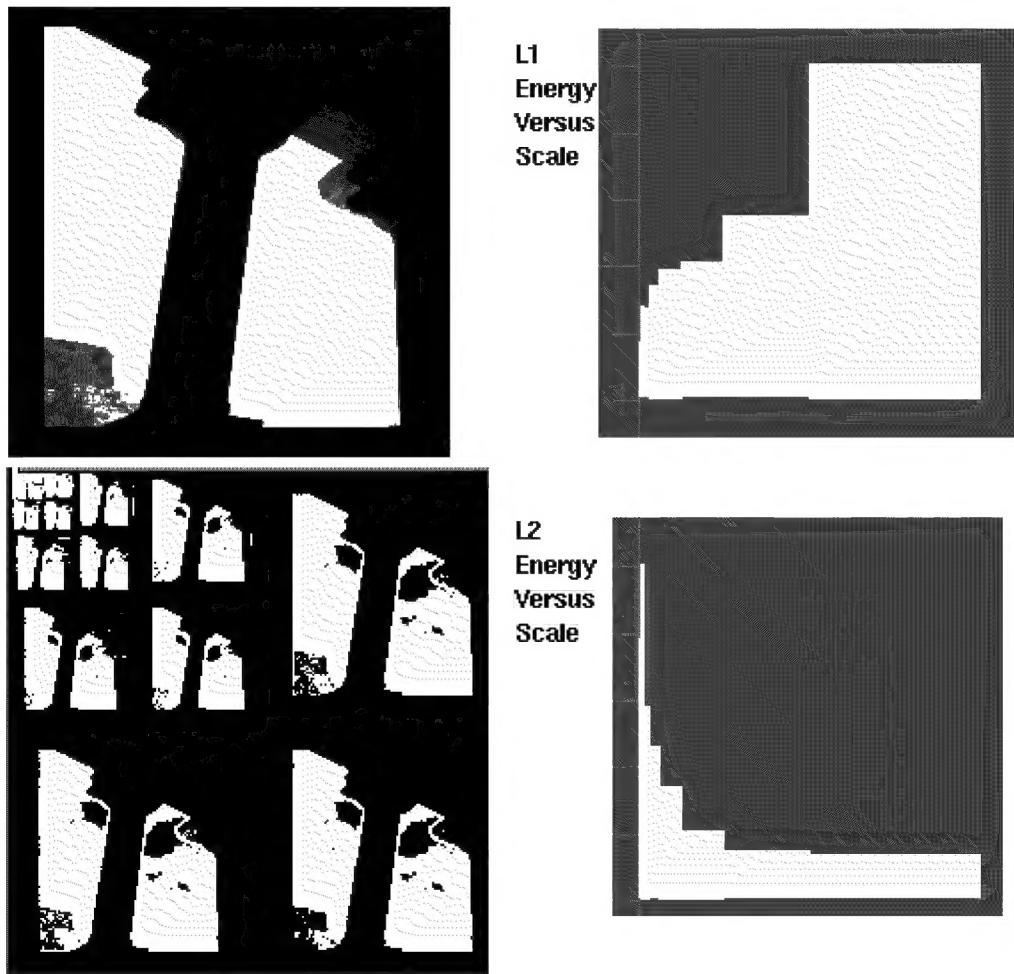


Figure 3. An image and its decomposition into orthonormal Daubechies wavelets (Mallat decomposition) are presented. The  $L_p$  energy of the wavelet coefficients as a function of scale provides an estimate of the Besov membership. The interpolation  $K$  functor, a measure of the difference in smoothness between object and background, is provided locally in terms of wavelets coefficients as the segmentation proceeds from coarse to fine segmentation.